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HIGH-MULTIPLICITY PROCESSES

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We wish to demonstrate that investigation of asymptotically high multiplicity (AHM) hadron reactions may solve, or at least clear up, a number of problems unsolvable by other ways. We would lean upon the idea: (i) the reactions final state entropy is proportional to multiplicity and, by this reason, just in the AHM domain one may expect the *equilibrium* final state and (ii) the AHM final state is *cold* because of the energy-momentum conservation laws. This means that the collective phenomena may become important in the AHM domain. The possibility of hard *processes* dominance is considered also.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

Процессы с большой множественностью

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Мы хотим показать, что исследование реакций рождения асимптотически большой множественности (АБМ) адронов может решить или, по крайней мере, прояснить, некоторые проблемы, неразрешимые другим образом. Мы собираемся следовать идее: (i) энтропия конечного состояния пропорциональна множественности, и поэтому именно в области АБМ можно ожидать образования равновесного состояния, (ii) конечное состояние с АБМ должно быть холодным в силу сохранения энергии-импульса. Последнее может означать, что в области АБМ коллективные явления могут быть существенны. Рассматривается также вклад жестких процессов при рождении состояния с АБМ.

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1. INTRODUCTION

The interest to multiple production processes noticeably falls down during last decades and this trend is understandable. Indeed, it is hard to expect a new physics if (a) the processes are too complicated because of a large number of involved degrees of freedom and (b) there is not quantitative hadrons theory to describe the nonperturbative effect of color charge confinement.

It may be surprising at first glance that the asymptotically high multiplicity (AHM) processes are simple at least from theoretical point of view [1][2]. Nevertheless this is so and

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[2]The AHM triggering problem will not be considered. It will be considered in subsequent paper, see also [2].

our aim is to demonstrate this suggestion. This follows from expectation that the fluctuations (quantum as well) are small considering AHM final states creation[3].

We wish to note firstly that the AHM final states are equilibrium since the corresponding entropy reach a maximum in the AHM region [3]. Then a comparatively small number of simplest parameters should be needed for AHM states description because of the correlations relaxation principle of Bogoliubov[4].

Besides, the AHM processes may become hard and, therefore, we may neglect (with exponential accuracy) the background low-p_t nonperturbative channel of hadrons production. Intuitively this suggestion is evident noting that in all soft Regge-like theories, with interaction radii of hadrons $\sim \sqrt{\ln s}$, one cannot 'stir' in the disk of area $\sim \ln s$ arbitrary number of partons, since the high-p_t cutoff. Then the Regge-like theories are unable to describe the multiplicities $n > \ln^2 s$, if each parton in the disk is a source of $\sim \ln s$ hadrons. At high energies the AHM domain is sufficiently wide ($\ln^2 s << n_{\max} \sim \sqrt{s}$) and, therefore, should be 'occupied' by hard processes [1]. Indeed, the mean transfer momentum of created particle grows with n_c, see [4].

So, the interest to AHM processes seems evident, since: (a) being hard, the experimental investigation of them may help to clear up the structure of fundamental Lagrangian, and (b) the theoretical predictions in this field are more or less clear in the asymptotically free theories.

It seems constructive to start investigation of hadron dynamics from AHM domain and, experienced in this field, go to the moderate multiplicities. The to-day experimental status of AHM is following. There is experimental data at TeVatron energies up to $n_c \approx 5\bar{n}_c(s)$ [4]. The cross section $\sigma_{n_c} \approx 10^{-5}\sigma_{\rm tot}$ at this values of charged particles multiplicity n_c. The events with higher multiplicity are unknown, if the odd cosmic data is not taken into account. The main experimental problem to observe AHM is smallness of corresponding cross sections and impossibility to formulate experimentally the 'naive' trigger tagging the AHM events by charged particles multiplicity[5]. We think that this question is very important since otherwise the AHM problem stay pure academic.

In Sec.2 we will give the physical interpretation of possible asymptotics over n. In Sec.3 the hard Pomerons contribution in AHM domain is discussed. In Sec.4 we will describe the QGP formation signal at AHM.

2. CLASSIFICATION OF ASYMPTOTICS OVER n

It is better to start from pure theoretical problem assuming that the incident total CM energy $E = \sqrt{s}$ is arbitrarily high and to consider asymptotics over the total multiplicity n, assuming nevertheless that $n << n_{\max} = \sqrt{s}/m$, where $m \simeq 0.2$ Gev is the characteristic

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[3]The space density of colored constituents becomes large and 'cold' in the AHM domain.

[4]In this case, following the ergodic hypothesis, one may restrict oneself by event-by-event measurements, i.e. it is sufficient to have a small number of AHM events. This seems important from experimental point of view since the cross sections in the AHM domain are assumed extremely small.

[5]We suggest to reject tagging the event by the multiplicity noting that it is enough to have the 'cold' final state in the AHM domain [2]. Moreover, formulating the theory we would try to generalize ordinary inclusive approach [5].

hadron mass, to exclude the influence of phase space boundaries. Last one means that in the sum

$$\Xi_{\max}(z,s) = \sum_{n=1}^{n_{\max}} z^n \sigma_n(s)$$

we should choose the real positive z so small that upper boundary is not important and we can sum up to infinity:

$$\Xi(z,s) = \sum_{n=1}^{\infty} z^n \sigma_n(s). \tag{2.1}$$

This trick allows one to introduce classification of σ_n asymptotics counting the singularities of $\Xi(z,s)$ over z [1]. So, if z_c is the solution of equation:

$$n = z\frac{\partial}{\partial z} \ln \Xi(z,s), \tag{2.2}$$

then in the AHM region

$$\sigma_n \sim e^{-n \ln z_c(n,s)}. \tag{2.3}$$

To do further step we will use the connection with statistical physics [6]. By definition

$$\sigma_n(s) = \int d\omega_n(q)\delta(p_a + p_b - \sum_{i=1}^{n} q_i)|A_n|^2, \tag{2.4}$$

where A_n is the $a+b \to$ (n hadrons) transition amplitude and $d\omega_n(q)$ is the n particles phase space element. There is a well-known in the particle physics trick [7] as this ($3n$)-dimensional integrals may be calculated. For this purpose one should use the Fourier transformation of the energy-momentum conservation δ-function. Then, in the CM frame, if n is large,

$$\sigma_n(s) = \int \frac{d\beta}{2\pi} e^{\beta E} \rho_n(\beta), \tag{2.5}$$

where

$$\rho_n(\beta) = \int d\omega_n(q) \prod_{i=1}^{n} e^{-\beta\varepsilon_i} |A_n|^2, \tag{2.6}$$

and ε_i is the i-th particles energy.

Obviously this trick is used to avoid the constraints from the energy-momentum conservation δ-function. But it has more deep consequence. So, if we consider interacting particles a and b in the black-body environment, then we should use the occupation number $\bar{n}_{ext}$ instead of 'Boltzmann factor' $e^{-\beta\varepsilon_i}$. For bosons

$$\bar{n}_{ext}(\beta\varepsilon) = \{e^{\beta\varepsilon} - 1\}^{-1}. \tag{2.7}$$

In result, replacing $e^{-\beta\varepsilon_i}$ by $\bar{n}_{ext}$,

$$\Xi(\beta, z) = \sum_{n} z^n \rho_n(\beta) \tag{2.8}$$

would coincide *identically* with big partition function of relativistic statistical physics, where β is the inverse temperature $1/T$ and z is the activity: the chemical potential $\mu = T \ln z$ [8].

We may use this S-matrix interpretation of statistical physics and consider $\Xi(z,s)$ as the energy representation of the partition function. Then, following Lee and Yang [9], $\Xi(z,s)$ should be regular function of z, for practically arbitrary interaction potentials, in the interior of unite circle. This means, using estimation (2.3), that

$$\sigma_n(s) < O(1/n), \tag{2.9}$$

i.e., $\sigma_n(s)$ should decrease faster than any power of $1/n$.

First singularity may locate at $z=1$. It is evident from definition (2.1) that $\Xi(z,s)$ would be singular at $z=1$ iff

$$\sigma_n(s) > O(e^{-n}), \tag{2.10}$$

i.e., in this case $\sigma_n(s)$ should decrease slower than any power of e^{-n}. It follows from estimation (2.3), in this case $z_c(n,s)$ should be the decreasing function of n. Note that such solution is, at first glance, impossible. Indeed, by definition (2.1), $\Xi(z,s)$ should be increasing function of z since all σ_n are positive. Then the solution of Eq.(2.2) should be increasing function of n. But, nevertheless, such possibility exists.

Remember for this purpose connection of z with chemical potential, $z_c(n,s)$ defines the work needed for creation of additional particle. So, $z_c(n,s)$ may be the decreasing function of n iff the vacuum is unstable and the transition from false, free from colorless particles (or being chiral invariant, etc.), vacuum to the true one means colorless particles (or of the chiral-invariance broken states, etc.) creation. Just this case corresponds to the first order phase transition [9, 10].

This phenomenon describes expansion of the domain of new phase with accelerating expansion of the new phase domains boundary, if the radius of domain is larger than some critical value [11]. So, $z_c(n,s)$ should decrease with n since z is conjugate to physical particles number in the domain of new phase.

The following singularity in the equilibrium statistics may locate at $z=\infty$ only. This is the general conclusion and means, as follows from (2.3), that

$$\sigma_n(s) < O(e^{-n}). \tag{2.11}$$

In this case $\sigma_n(s)$ should fall down faster than any power of e^{-n}. Note, investigation of the Regge-like theories gives just this prediction [12].

But considering the process of particles creation it is too hard to restrict oneself by equilibrium statistics. The final state may be equilibrium, as is expected in the AHM domain, but generally we consider the process of incident (kinetic) energy dissipation into particles mass. At very high energies having the AHM final state we investigate the process of highly nonequilibrium states relaxation into equilibrium one.

Such processes have interesting property readily seen in the following well-known model. So, at the very beginning of this century couple P. and T.Euhrenfest had offered a model to visualize Boltzmann's interpretation of irreversibility phenomena in statistics [13]. The model is extremely simple and fruitful. It considers two boxes with $2N$ numerated balls. Choosing number $l = 1, 2, ..., 2N$ *randomly* one must take the ball with label l from one box and put it to another one. Starting from the highly 'nonequilibrium' state with all balls in one box it is seen a tendency to equalization of balls number in the boxes. So, there is seen irreversible flow toward preferable (equilibrium) state. One can hope that this model reflects a physical reality of nonequilibrium processes with initial state very far from equilibrium. A theory of such processes with (nonequilibrium) flow toward a state with maximal entropy should be sufficiently simple to give definite theoretical predictions since there is not statistical fluctuation of this flaw.

Following this model one can expect total dissipation of incident energy into particle masses. In this case the mean multiplicity $\bar{n}$ should be $\simeq n_{\max}$ [3]. But it is well known that experimentally $\bar{n} << n_{\max}$. Explanation of this phenomena is hidden in the constraints connected with the conservation laws of Yang-Mills field theory. It is noticeable also that this constraints are not so rigid as in integrable systems, where there is no thermalization [14]. In the AHM domain we expect the free from above constraints dynamics. So, the AHM state may be the result of dissipation process governed by 'free' (from QCD constraints) irreversible flow.

The best candidates for such processes are stationary Markovian ones. They will be described by the so-called logistic equation [15] and lead to inverse binomial distribution with generating function

$$\Xi(z,s) = \sigma_{\rm tot}(s)\left\{\frac{z_s(s)-1}{z_s(s)-z}\right\}^{\gamma}, \quad \gamma > 0. \tag{2.12}$$

The normalization condition $\partial \ln \Xi(z,s)/\partial z|_{z=1} = \bar{n}_j(s)$ determines the singularity position:

$$z_s(s) = 1 + \gamma/\bar{n}_j(s). \tag{2.13}$$

Note, $z_s(s) \to 1$ at $s \to \infty$ since $\bar{n}_j(s)$ should be increasing function of s.

Mostly probable values of z tend to z_s from below with rising n:

$$z_c(n,s) \simeq z_s - \gamma/n = 1 + \gamma\left(\frac{1}{\bar{n}_j(s)} - \frac{1}{n}\right). \tag{2.14}$$

This means that the vacuum of corresponding field theory should be stable. It is evident that σ_n decrease in this case as the $O(e^{-n})$:

$$\sigma_n \sim e^{-\gamma n/\bar{n}_j}. \tag{2.15}$$

The singular solutions of (2.12) type arise in the field theory, when the s-channel cascades (jets) are described [16]. By definition $\Xi(z,s)$ coincide with total cross section at $z = 1$. Therefore, nearness of z_c to one defines the significance of corresponding processes. It follows from (2.14) that both s and n should be high enough to expect the jets creation. But the necessary condition is closeness to one of z_s, i.e., the high energies are necessary, and high n simplifies only their creation. Note the importance of jet creation processes in early Universe, when the energy density is extremely high.

Expressing the 'logistic grows law' the singular structure (2.12) leads to the following interesting consequence [15]. The energy conservation law shifts the singularity to the right. For instance, the singularity associated with two-jets creation is located at $z_c^{(2)}(s) = z_c(s/4) > z_c(s)$. Therefore, the multi-jet events will be suppressed with exponential accuracy in the AHM domain if the energy is high enough, since at 'low' energies even the exponential accuracy may be insufficient for such conclusion [1]. One may assume that there should be the critical value of incident energy at which this phenomena may realized. So the AHM are able to 'reveal' the jet structure iff the energies are high enough. In this sense the AHM domain is equivalent of asymptotic energies.

Summarizing above estimations we may conclude that

$$O(e^{-n}) \leq \sigma_n < O(1/n), \tag{2.16}$$

i.e., the soft Regge-like channel of hadron creation is suppressed in the AHM region in the high energy events with exponential accuracy.

3. HARD POMERON

During last 50 years the contribution (Pomeron) which governs the s-asymptotics of the total cross section $\sigma_{\text{tot}}(s)$ is to stay unsolved. The efforts in the pQCD frame show that the t-channel ladder diagrams from dressed gluons may be considered as the dynamical model of the Pomeron [17].

The BFKL Pomeron arises in result of summation, at least in the LLA, of ladder gluon diagrams in which the virtualities of space-like gluons rise to the middle of the ladder. To use the LLA these virtualities should be high enough. Noting that the 'cross-beams' (time-like gluons) of the ladder are the sources of jets it is natural that in the AHM domain the jet masses q_i^2 are large enough and one can apply the LLA.

Consideration of Pomeron as the t-channel localized object allows one to conclude that the multi-Pomeron contribution is $\sim 1/k!$ if k is the number of Pomerons. This becomes evident noting that in the t channel the distribution over k localized uncorrelated 'particles' should be Poissonian. This factorial damping should be taken into account in the AHM domain.

Following our above derived conclusion, number of 'cross-beams' (jets) should decrease with increasing n, and, therefore, in the AHM domain the BFKL Pomeron, with exponential accuracy, should degenerate into ladder with two 'cross-beams' only. The virtuality of time-like gluons becomes in this case $\sim \sqrt{s}$. So, the bare gluons are involved at high energies at the AHM. This solution is in agreement with our general proposition that in the AHM domain the particles creation process should be stationary Markovian.

We conclude that the AHM processes gave unique possibility to understand as the BFKL Pomeron is built up. But there is the problem, connected with masslessness of gluons. So, the vertices of time-like gluons emission are singular at the $q_i^2 = 0$. It is the well-known low-x problem. In the BFKL Pomeron this singularity is canceled by attendant diagrams of the 'real' soft gluons emission. However, this mechanism should be destroyed when number of created particles (i.e., of gluons) is fixed. Note, the solution of this problem is unknown.

We hope to avoid this problem noting that the heavy jets creation dominates in the AHM domain. This idea reminds the way as the infrared problem is solved in the QED (The emission of photons with wave length much better than the dimension of measuring device is summed up to zero.) The quantitative realization of this possibility for QCD is in progress now.

4. QUARK GLUON PLASMA

There is a question: can we simulate in the terrestrial conditions the early Universe? The hot, dense, pure from colorless particles quark-gluon plasma (QGP) is the best candidate for investigation of this fundamental problem [18].

In our opinion [5] the plasma is a state of unbounded charges. The 'state' assumes presence of some parameters characterizing the collective of charges and the 'unbounded' assumes that the state is not locally, in some scale, neutral (we discuss the globally neutral plasma).

The ordinary QED plasma assumes that the mean energies of charged particles are sufficiently high (higher than the energy of particles acceptance), i.e., the QED plasma is 'hot'. The QCD plasma in opposite is not 'hot'(in corresponding energy scale) since the thermal motion moves apart the color charges. This leads to sufficient polarization and further 'boiling' of vacuum, with creation of $q\bar{q}$ pairs. So, the QCD plasma should be dense and at the same time 'cold' enough.

There is two principal possibilities to create such state. Mostly popular is QGP plasma formation in the heavy ion-ion collisions at high energies. It is believed that at the central (head-on) collisions one can observe the QGP in the CM central region of rapidities. But there are not in to-day situation the unambiguous (experimental) signals of QGP formation (number of the theoretical possibilities are discussed in literature).

Other possibility opens the AHM region: since in this region the hard channel of particles creation is favourable dynamically (at high energies), one can try to consider the collective of color charges on preconfinement stage as the plasma state. Because of energy-momentum conservation this state would be 'cold'. We should underline that possible cold QGP (CQGP) formation is just the dynamical, nonkinematical, effect: the estimation (2.16) means that the sufficient polarization of vacuum and its 'boiling' effects are insufficient, are frozen, at the AHM. So, considered CQGP remains relativistic. This solves the problem of unbounded charges formation[6].

But the question – may we consider the collective of colored charges created in the AHM events as the 'state' – remains open. To-day situation in theory is unable to give answer to this question (even in the QCD frame, this will be discussed). But we can show the experimentally controlled condition when the same parameters may be used to characterize this collective.

For instance, we may examine in what conditions the mean energy of colored particles may be considered as such parameter. Let us return to the definition (2.5) for this purpose. To calculate the integral over β we will use the stationary phase method. Mostly probable values of β are defined by equation of state:

$$E = \partial \ln \rho(\beta, z)/\partial\beta. \tag{4.17}$$

It is well known that this equation has positive real solution β_c. Then, as was noted above, β_c coincides with inverse temperature $1/T$ and this definition of T is obvious in the micro-canonical formalism of statistical physics.

[6] To amplify this effect it seems reasonable to create AHM in the ion-ion collisions

The parameter β_c is 'good', i.e., has a physical meaning, iff the fluctuations near it are Gaussian (It should be underlined that the value of fluctuations may be arbitrary, but the distribution should be Gaussian). This is so if, for instance,

$$\frac{\rho^{(3)}}{\rho} - 3\frac{\rho^{(2)}\rho^{(1)}}{\rho} + 2\frac{(\rho^{(1)})^3}{\rho} \approx 0, \tag{4.18}$$

where, for identical particles,

$$\rho^{(k)}(\beta_c, z) \equiv \frac{\partial^k \rho(\beta_c, z)}{\partial \beta_c^k} =$$

$$(-z\frac{\partial}{\partial z})^k \int d\omega_n(q) \prod_{i=1}^{k} \varepsilon(q_i) f_k(q_1, ..., q_k; \beta_c, z) \tag{4.19}$$

and $f_k(q_1, ..., q_k; \beta_c, z = 1)$ is the k-particle inclusive cross section. Therefore, the (4.18) condition requires smallness of energy correlation functions. It is the obvious in statistics energy correlations relaxation condition near the equilibrium. One can find easily the same condition for higher correlation functions.

This conditions establish the equilibrium, when knowledge of one parameter (β_c in considered case) is enough for whole systems description. The analogous conditions would arise if other parameters are considered. For instance, the (baryon, lepton, etc.) charge correlations relaxation condition means the 'chemical' equilibrium. The quantitative expression of this phenomena is smallness of corresponding correlation functions.

This conditions are controllable experimentally. But it is hard to expect that at finite values of n, where the cross sections are not too small, above derived conditions are hold, even in the AHM region. Later we will find more useable from experimental point of view conditions making more accurate analyses.

5. CONCLUSION

The AHM events offer interesting possibility of investigating the phase transition phenomena. Noting that the system has a tendency to become equilibrium in the AHM domain and noting that the Gibbs free energy is $\sim \ln \Xi$, we can compare the heat capacity in the hadrons and photons (or leptons) systems in the AHM domain. This will be the direct measurement of phase transition.

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